UNIVERSITY OF TECHNOLOGY SYDNEY
Faculty of Engineering and Information Technology

Distillation and Simulation in Quantum Information

by

Kun Fang

A Thesis Submitted
in Partial Fulfillment of the
Requirements for the Degree

Doctor of Philosophy

Supervisor: Prof. Runyao Duan
Co-supervisor: Prof. Mingsheng Ying

Sydney, Australia

2018

Certificate of Original Authorship

I certify that the work in this thesis has not been previously submitted for a degree nor has it been submitted as a part of the requirements for other degree except as fully acknowledged within the text.

I also certify that this thesis has been written by me. Any help that I have received in my research and in the preparation of the thesis itself has been fully acknowledged. In addition, I certify that all information sources and literature used are quoted in the thesis.

Production Note:
Signature removed prior to publication.

Signature: ____________

Date: 17/09/2018

Acknowledgements

I am very fortunate to meet lots of brilliant people during my PhD studies. Little of the work in this thesis would have been possible without their continuous support.

First and foremost, I would like to thank my supervisor Runyao Duan for his patient guidance and supervision during my research. I am deeply grateful for his inspiring discussions and thank him for encouraging me to discuss and collaborate with others. I would like to show my greatest appreciation to my co-supervisor Mingsheng Ying for his willingness to provide me with some insightful suggestions and keep reminding me to broaden my research areas.

A special thanks goes to my mentor Marco Tomamichel, who generously set aside time to answer my questions and patiently guide me in the entropy zoo. I can always learn something new after discussing with him. I also would like to send my special thanks to my friend and frequent collaborator Xin Wang for our innumerable discussions. Thanks him for teaching me the powerful tool of semidefinite programming and some practical techniques for doing numerical experiments.

Most of my work has been collaborative, and I am grateful to all my collaborators for inspiring discussions and productive collaborations. I would like to thank all my co-authors: Gerardo Adesso, Mario Berta, Runyao Duan, Xiaoyu He, Ludovico Lami, Bartosz Regula, Xiaoming Sun, Marco Tomamichel, Xin Wang and Wei Xie for many stimulating discussions, helpful advice and well-appreciated support. Besides the colleagues I have mentioned so far, I want to further extend my gratitude to Charles Bennett, Matthias Christandl, Omar Fawzi, Gilad Gour, Ji Guan, Masahito Hayashi, Yinan Li, Laura Mančinska, Alexander Müller-Hermes, Dong Yang for many valuable discussions.

I am grateful to the Centre for Quantum Software and Information at the University of Technology Sydney for providing an excellent research environment including a number of travel opportunities for graduate students. I also thank all the centre members creating such a pleasant and productive atmosphere. In the course of my PhD studies, I always had a great time and benefited a lot from my academic travels. In particular, I want to thank Gerardo Adesso, Mario Berta, Matthias Christandl, Xiaoming Sun for their hospitable host during my visits.

I would like to express my sincere thanks to all my friends. Particular thanks goes to Yinan Li and Xin Wang for their accompany. Finally, I want to save my deepest thanks to my family for their unconditional love and support.

Abstract

We use the techniques of convex optimization, especially semidefinite programming, to study two kinds of fundamental tasks, i.e., distillation and simulation in quantum information theory. We investigate these tasks in a unified framework of resource theory and focus on their computation and characterization with finite resources. Particularly we study the tradeoff among relevant parameters such as the number of resource copies, resource transformation rate, error tolerance and success probability.

In the first part, we study the task of distillation for two different resources, maximally entangled state and maximally coherent state, representing nonlocal and local "quantumness" respectively. For entanglement distillation, we derive an efficiently computable second-order estimation of the distillation rate for general quantum states, which are tight for quantum states of practical interest. Our study overcomes the limitation of conventional research either focusing on the asymptotic rate or ignoring the computability. For the coherence distillation, we perform finite analysis for both deterministic and probabilistic scenarios. Our results unveil several new features of coherence from a resource theoretic viewpoint and contribute to an increased understanding of the fundamental properties of different sets of free operations.

In the second part, we investigate the resource cost of simulating a quantum channel via quantum coherence or another quantum channel. We introduce the channel's analogs of max-relative entropy, logarithmic robustness and max-information of quantum states, providing their operational interpretation with the channel simulation cost via different resources. Particularly, we establish the asymptotic equipartition property of the channel's max-information, that is, it converges to the quantum mutual information of the channel in the independent and identically distributed asymptotic limit. As applications, this asymptotic equipartition property implies the quantum reverse Shannon theorem in the presence of non-signalling correlations.

From the perspective of resource theory, the worth of a resource can usually be characterized by the minimum distance to a set of useless resources under a proper distance measure. We give such characterization for all the tasks studied in this thesis, and find that the distance measure for the distillation and simulation process naturally corresponds to the quantum hypothesis testing relative entropy and the max-relative entropy, respectively.

List of Publications

During the time of my PhD study, I am honored to collaborate with many excellent researchers. Parts of this thesis are based on material contained in the following papers.

- **K. Fang**, X. Wang, M. Tomamichel, and R. Duan, *Non-asymptotic entanglement distillation*, submitted, arXiv:1706.06221, 2017. [Chapter 3]
- B. Regula, **K. Fang**, X. Wang, and G. Adesso, *One-shot coherence distillation*, Physical Review Letters 121, 010401, 2018. [Chapter 4]
- **K. Fang**, X. Wang, L. Lami, B. Regula, G. Adesso, *Probabilistic distillation of quantum coherence*, Physical Review Letters 121, 070404, 2018. [Chapter 4]
- **K. Fang**, X. Wang, M. Tomamichel, and M. Berta, *Quantum channel simulation and the channel's max-information*, submitted, arXiv:1807.05354, 2018. [Chapter 5]
- M. Díaz, **K. Fang**, X. Wang, M. Rosati, M. Skotiniotis, J. Calsamiglia, A. Winter, *Using and reusing coherence to realize quantum processes*, submitted, arXiv:1805.04045, 2018. [Chapter 5]

Other work on which this manuscript does not focus:

- W. Xie, **K. Fang**, X. Wang, and R. Duan, *Approximate broadcasting of quantum correlations*, Physical Review A 96, 022302, 2017.
- X. Wang, **K. Fang**, and M. Tomamichel, *On converse bounds for classical communication over quantum channels*, submitted, arXiv:1709.05258, 2017.
- X. Wang, **K. Fang**, and R. Duan, *Semidefinite programming converse bounds for quantum communication*, submitted, arXiv:1709.00200, 2017.
- X. He, **K. Fang**, X. Sun, and R. Duan, *Quantum advantages in Hypercube game*, arXiv: 1806.02642, 2018.

Contents

List of Figures

Abbreviations and Notations

Abbreviation	Description
CPTP	Completely positive trace-preserving map
SDP	Semidefinite program
i.i.d.	Independent and identically distributed
1-LOCC	Local operations and one-way classical communication
LOCC	Local operations and classical communication
SEP	Separable
PPT	Positive partial transpose preserving
NS	Non-signalling
MIO	Maximally incoherent operation
DIO	Dephasing-covariant incoherent operation
IO	Incoherent operation
SIO	Strictly incoherent operation
PIO	Physical incoherent operation
AEP	Asymptotic equipartition property
QRST	Quantum reverse Shannon theorem
l.h.s.	Left-hand side of an equation
r.h.s.	Right-hand side of an equation
min	minimize
max	maximize
s.t.	subject to

Table 1 : A list of abbreviations.

Notation	Description
A, B', AB	Typical physical systems and joint systems
$\mathcal{H}_A$	Hilbert space over system A
d_A, $\|A\|$	Dimension of the system A
$\mathcal{S}_=(A)$	The set of normalized quantum states on $\mathcal{H}_A$
$\mathcal{S}_\leq(A)$	The set of sub-normalized quantum states on $\mathcal{H}_A$
SEP(A:B)	The set of separable states shared between A and B
PPT(A:B)	The set of PPT states shared between A and B
PPT′(A:B)	The Rains set
$\mathcal{I}$	The set of incoherent quantum states
$\mathcal{I}_H$	The set of diagonal Hermitian operators
ρ, σ, ω	Typical quantum states or subnormalized states
$\|\varphi\rangle, \varphi$	Typical pure states, i.e., rank-1 quantum states
$\Phi_{AA'}$, Φ_k	Maximally entangled state (with local dimension k)
Ψ_k	Maximally coherent state (with dimension k)
Tr_A	Partial trace over system A
id_A	Quantum noiseless channel on the space $\mathcal{H}_A$
$F(\rho, \sigma)$	Generalized fidelity between ρ and σ
$P(\rho, \sigma)$	Purified distance between ρ and σ
$\log \equiv \log_2$	Logarithm in base 2
$\ln$	Natural logarithm
$\mathcal{L}(A)$, $\mathcal{L}(A, B)$	The set of linear operators on $\mathcal{H}_A$ and from $\mathcal{H}_A$ to $\mathcal{H}_B$
$\mathcal{P}(A)$	The set of positive semi-definite operators on $\mathcal{H}_A$
$\mathcal{H}erm(A)$	The set of Hermitian operators on $\mathcal{H}_A$
$X \geq Y$	Equivalent to $X - Y \in \mathcal{P}(\mathcal{H})$
$X^\dagger$	The adjoint operator of X
$\overline{X}$	The complex conjugate of X
X^T, X^{T_A}	The transpose and partial transpose of X
$\mathbb{1}_A$	Identity operator on Hilbert space $\mathcal{H}_A$
$\mathcal{E}, \mathcal{F}$	Typical completely positive and trace-preserving maps
Π	Typical free quantum operations (codes)
Ω	Typical set of free operations (codes)
Δ	Completely dephasing channel (diagonalizing map)
$\mathcal{N}_{A\to B}, \mathcal{M}_{A\to B}$	Typical quantum channels from $\mathcal{L}(A)$ to $\mathcal{L}(B)$
$J_\mathcal{N}$	Choi-Jamiołkowski operator of the operation $\mathcal{N}$
$\\|X\\|_1$	Trace norm, i.e., the sum of all singular values of X
$\\|X\\|_\infty$	Operator norm, i.e., the largest singular value of X
$\\|\mathcal{E}\\|_\diamond$	Diamond norm (Completely bounded trace norm)

Table 2 : A list of basic notations.